

Given: Data for one operating condition of a pump test

Measured values: $h = 30 \text{ in}$, $m = 500 \text{ g}$, $\Delta t = 30 \text{ s}$

Pump voltage = 12V, pump current = 1A

Physical constants:

$d = \frac{3}{16} \text{ inch} = \text{I.D. of tubing}$

$\rho = 1000 \text{ kg/m}^3 = \text{density of water}$

Analysis:

1. Compute velocity of water

$$\text{mass flow rate} = \dot{m} = \rho A v \Rightarrow v = \frac{\dot{m}}{\rho A}$$

$A = \text{cross-sectional area of the tubing} = \frac{\pi}{4} d^2$

$v = \text{average velocity}$

$$\text{From measurements: } \dot{m} = \frac{\text{mass collected}}{\text{time of collection}} = \frac{m}{\Delta t}$$

$$v = \frac{\dot{m}}{\rho A} = \frac{m}{\Delta t} \frac{1}{\rho} \frac{4}{\pi d^2}$$

$$= \underbrace{\frac{500 \text{ g}}{30 \text{ s}} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{1}{1000 \text{ kg/m}^3}}_{0.0167 \frac{\text{kg}}{\text{s}}} \times \frac{4}{\pi \left(\frac{3}{16} \text{ inch} \times \frac{25.4 \text{ mm}}{\text{inch}} \times \frac{1 \text{ m}}{1000 \text{ mm}} \right)^2} \underbrace{\frac{1}{1.781 \times 10^{-5} \text{ m}^2}}_{= \frac{1}{A}}$$

$$v = 0.936 \frac{\text{m}}{\text{s}}$$

Analysis, Continued

2. Calculate volumetric flow rate

$$Q = vA = \frac{m}{\Delta t} \frac{1}{s}$$

$$= (0.936 \frac{m}{s}) \left[\underbrace{\frac{\pi}{4} \left(\frac{3}{16} \text{inch} \times 25.4 \frac{\text{mm}}{\text{inch}} \times \frac{1 \text{m}}{1000 \text{mm}} \right)^2}_{A = 1.781 \times 10^{-5} \text{ m}^2} \right]$$

$$Q = 1.667 \times 10^{-5} \frac{\text{m}^3}{\text{s}} \times \frac{1 \text{L}}{1000 \text{cm}^3} \times \left(\frac{100 \text{cm}}{\text{m}} \right)^3 \times \frac{60 \text{s}}{\text{min}} = 1.00 \frac{\text{L}}{\text{min}}$$

$$\boxed{Q = 1.667 \times 10^{-5} \frac{\text{m}^3}{\text{s}} = 1.00 \frac{\text{L}}{\text{min}}}$$

3. Compute Efficiency

$$\eta = \frac{\text{Energy Output}}{\text{Energy Input}} = \frac{\text{Power Output}}{\text{Power Input}} = \frac{mgh + \frac{1}{2} \dot{m}V^2}{VI} = \frac{\dot{m}(gh + \frac{V^2}{2})}{VI}$$

$$\dot{m} = \frac{m}{\Delta t} = \frac{500 \text{g}}{30 \text{s}} \times \frac{1 \text{kg}}{1000 \text{g}} = 0.0167 \frac{\text{kg}}{\text{s}}$$

$$\eta = \frac{\left(0.0167 \frac{\text{kg}}{\text{s}}\right) \left[9.8 \frac{\text{m}}{\text{s}^2} \times 30 \text{inch} \times 25.4 \frac{\text{mm}}{\text{inch}} \times \frac{1 \text{m}}{1000 \text{mm}} + \frac{1}{2} (0.936 \frac{\text{m}}{\text{s}})^2 \right]}{(12 \text{V})(1 \text{A})}$$

$$= \frac{\left(0.0167 \frac{\text{kg}}{\text{s}}\right) \left(7.468 \frac{\text{m}^2}{\text{s}^2} + 0.438 \frac{\text{m}^2}{\text{s}^2} \right)}{12 \text{W}}$$

$$= \frac{0.132 \text{W}}{12 \text{W}}$$

$$= 0.011$$

$$\therefore \boxed{\eta = 1.1\%}$$